A Survey on the Effect of Small Snapshots Number and SNR on the Efficiency of the MUSIC Algorithm

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***Abstract*—The Multiple Signal Classification (MUSIC) algorithm is applied for a number of cases, in order to associate the performance of angular resolution of a Uniform Linear Array (ULA) with small number of snapshots and Signal-to-Noise Ratio (SNR). The discovery of low limits of these parameters is essential for the practical implementation of MUSIC. The resolution dependence is presented in diagrams and the results are compared with corresponding theoretical limits.**

I. Introduction

The problem of distinguishing multiple uncorrelated sources, that impinge on an array antenna, is known as Direction of Arrival (DOA) estimation. The solution to this problem serves radar and sonar applications, as well as wireless communications in general. Among other developed methods, the Multiple Signal Classification (MUSIC) algorithm offers high resolution and low computational complexity [1]. It is a method based on the eigenvalue decomposition of the correlation matrix of the inputs to an array. This specific algorithm is making use of the array steering vector and the array correlation matrix, and creates a pseudospectrum, which is the basis for angle estimations. The pseudospectrum is given by the following expression from [2]:

$$P_{MUSIC} = \frac{1}{a^H(\theta)\cdot E_N \cdot E_N^H \cdot a(\theta)} \tag{1}$$

where $\alpha(\theta)$ is the array steering vector, H is the operator for a Hermitian transpose vectro and E_N is the noise subspace, defined by the smallest eigenvalues of the array correlation matrix:

$$\overline{R}_{xx} = \overline{A}\overline{R}_{ss}\overline{A}^H + \overline{A}\overline{R}_{sn} + \overline{R}_{ns}\overline{A}^H + \overline{R}_{nn} \tag{2}$$

All four correlation matrices (signal, signal/noise, noise/signal and noise correlation matrix, respectively) of the above calculation are dependent on the number of snapshots *K*. The number of snapshots can be determined by adjusting a Digital Signal Processor (DSP) to control our array in a specific way. As K decreases, the processing system becomes faster and thus more practical. In this work, an effort is carried out in order to investigate the limit of K, until which we still get satisfactory results. The signal correlation matrix is additionally dependent on the Signal-to-Noise Ratio (SNR), which characterizes every propagation environment by providing a measure of signal strength versus noise. A is the matrix of array steering vectors $\alpha(\theta_i)$. For simplicity's sake only Uniform Linear Array (ULA) antennas are employed in this work. Each ULA has a characteristic resolution at its broadside, which is defined by the -3dB points of the main lobe of the radiation pattern, as shown in Fig.1:

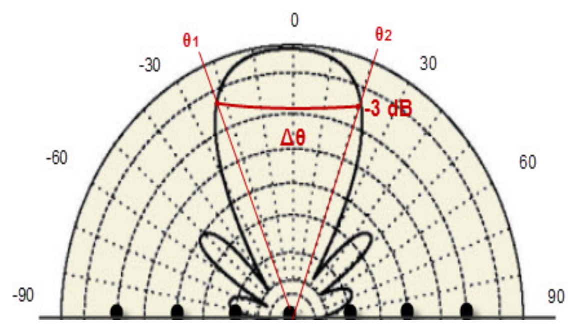

Figure 1. The ULA configuration and definition of its broadside resolution, *Δθ*.

The broadside resolution depends on the number of array elements, *M*, as described in the equation below [3]:

$$\Delta\theta^\circ = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.392\lambda}{\pi M d}\right)\right] \tag{3}$$

where $d=dx=0.5\lambda$. The effect of the number of snapshots, as well as the effect of the SNR is presented below, from the application of the MUSIC algorithm, for ULA's with 3 elements ($\Delta\theta^o$=34.4°) and 10 elements ($\Delta\theta^o$=10.2°).

II. Simulation Results

Several simulations were made using code written in MATLAB [4]. The noise environment consists of identically distributed white Gaussian noise with variance σ^2 = 0.5. The presented diagrams concern cases with two uncorrelated sources, (the presence of more sources hardly affects the results), located symmetrically about the broadside direction. The behavior of the two ULAs with 3 and 10 equidistant elements was examined, and is shown in Figs. 2 and 3. For

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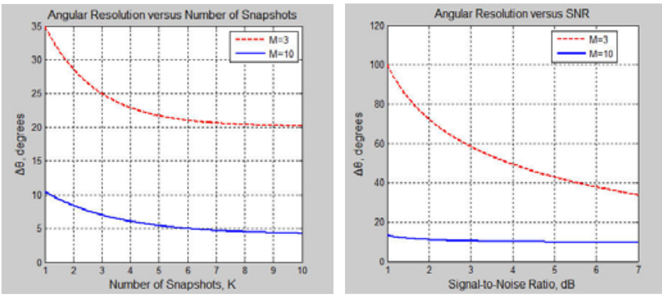
each ULA, the effect of the number of snapshots was studied for a fixed SNR = 6 dB, as shown in Fig. 2 (a). The snapshots number was defined to be $1 \leq K \leq 10$, in order to describe a relatively quick processing system. Next, simulation results for a constant number of snapshots ($K = 1$) and $1 \leq \text{SNR} \leq 6$ are depicted in Fig. 2 (b).

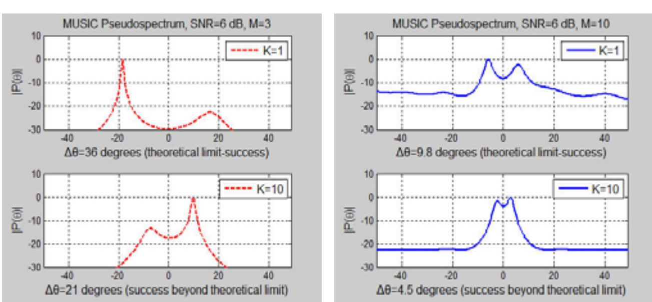


(a) (b)

Figure 2. Resolution (a) versus the number of snapshots for SNR= 6 dB and (b) versus SNR for K=1.

Although *K* is definitely an integer, the results in Fig. 2(a) are shown in a continuous curve so that the qualitative characteristics of the diagrams can be percepted. Fig. 2(a) shows clearly that the increase of the number of snapshots can improve the resolution of the antenna, while the theoretical limit seems to be corresponding to the case when SNR = 6 dB and $K = 1$. Moreover, the effect seems to be stronger for arrays with fewer elements. On the other hand, Fig. 2(b) indicates that the increase of the value of the SNR beyond 6 dB does not affect resolution in a considerable way.

The corresponding pseudospectrae are presented below for M=3 and M=10 (Fig. 3 (a) and 3 (b) respectively). The upper diagrams show that the theoretical resolution is achieved for K=1, while the lower diagrams indicate success of MUSIC algorithm beyond the theoretical limit for K=10. In other words, the array resolution is well improved by increasing the snapshots number.



(a) (b)

Figure 3. MUSIC Pseudospectrae for (a) M=3 and (b) M=10.

Tables I and II show numerical results derived from the implementation of MUSIC algorithm in contrast with theoretical limits from (3).

TABLE 1 THEORETICAL ANGULAR RESOLUTION VERSUS THE RESOLUTION WHEN *K* INCREASES (FIXED SNR=6 DB).

Theoretical Limit		MUSIC algorithm	K
M=3	Δθ=34.4º	35º	1
		25º	3
		20.2º	10
M=10	Δθ=10.2º	10.5º	1
		7º	3
		4.3º	10

TABLE 2 THEORETICAL ANGULAR RESOLUTION VERSUS THE RESOLUTION WHEN SNR INCREASES (FIXED *K*=1).

Theoretical Limit		MUSIC algorithm	SNR (dB)
M=3	Δθ=34.4º	100º	1
		50º	4
		36º	6
		35º	7
M=10	Δθ=10.2º	13.2º	1
		10º	4
		9.8º	6
		9.6º	7

III. CONCLUSIONS

The effect of having a small number of snapshots, *K*, and low SNR on the resolution of a ULA was investigated. The increase of *K* affects the ability of a ULA to distinguish at least two uncorrelated sources in a considerable way. On the other hand, the increase of SNR above about 6 dB can hardly improve the ULA resolution. On both cases, the effect appears to obey to an exponential in general. Results for the effect of low values of *K* and *SNR*, in the case of the application of the MUSIC algorithm in 2 dimensions [4, 5] are going to be shown, also, during the presentation.

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